DEVELOPMENT OF THERMOELASTIC TRANSMISSION CONDITIONS ACROSS A THIN INTERFACE.

Shubhra Pande[1,2],* , Andrea Piccolroaz[1], Séverine Romero Baivier[2]
*shubhra.pande@unitn.it

[1]University of Trento Via Mesiano, 77, 38123 Trento TN, Italy
[2] Vesuvius Group Rue de Douvrain 17, 7011 Ghlin, Belgium

1. ABSTRACT

Refractory devices composed of ceramics are employed wherever there is contact with molten metals as in crucibles filters, furnaces or systems for flow control. The mechanical properties of these materials are very different from metals.

The presentation focuses on basic equations related to thermal and mechanical transmission conditions across an interface between two different materials. These equations are linked to the development of material instabilities and crack propagation within the ceramics, in the view of application to refractories operating at high-temperature conditions in steel plants.

2. INTRODUCTION

We consider a coated spherical inclusion in a matrix, as shown in Fig.1. Consider a thin interphase of constant thickness *t* between two regions 1 and 2. The three regions are isotropic,elastic and homogeneous and are labelled as *i, 1* and *2*.

In 3 phase model, the elastic and thermal properties are considered to be perfect transmission conditions, as they are continuous across the interphase.

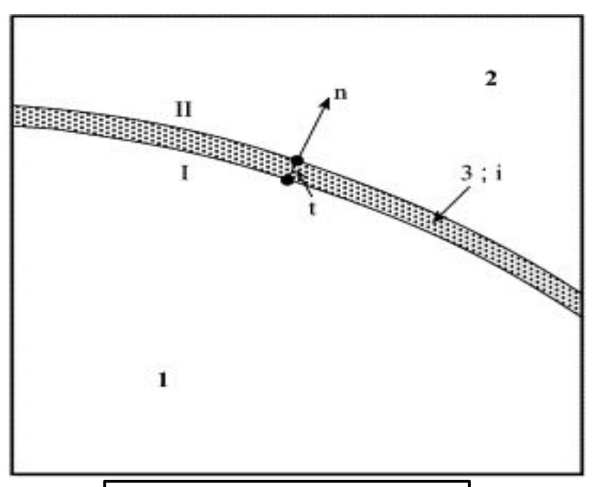

Fig.1: Interface condition

When the thickness of the interphase material is reduced, the properties are no longer continuous across the interphase, which is leads to the imperfect transmission conditions. They transform a three phase boundary value problem with perfect interfaces into a two phase problem with imperfect interface. The aim of the work presented here is to derive the thermal and elastic transmission conditions, across the interphase between two materials, for a spring-type interface. The results obtained will be extended to thermoelastic and thermoplastic materials.

3. PROBLEM FORMULATION

Elastic Subproblem

Governing Equation:

Constitutive Equation: $\underline{\sigma} = \lambda \mathbf{tr}(\underline{\varepsilon})\mathbf{I} + 2\mu\underline{\varepsilon}$

Across a perfect interface:

- displacement: $\underline{\mathbf{u}}$
- stress: $\underline{\mathbf{t}} = \underline{\sigma}.\underline{\mathbf{n}}$

} continuous

Transmission Conditions:

$$u_j^2(S_{\text{II}}) - u_j^1(S_{\text{I}}) = [u_j],$$
$$\sigma_{1j}^2(S_{\text{II}}) - \sigma_{1j}^1(S_{\text{I}}) = [\sigma_{1j}],$$

Thermal Subproblem

Governing Equation:

Fourier's Law: q = -k*(dT/dx)

Across a perfect interface,

- Temperature: T
- Heat flux: q

} continuous

Transmission Conditions:

$$[\varphi] = \varphi^i(a+t,\theta) - \varphi^i(a,\theta),$$
$$[q_n] = q_n^i(a+t,\theta) - q_n^i(a,\theta).$$

φ is the temperature

4. MATERIAL AND METHOD

A 3-phase problem consists of a coated sphere (material 1) of another material embedded in an infinite medium (material 2), as shown in Fig. 2 The coating of the sphere is of a finite thickness and is made of different material, other than 1 and 2, and is denoted as material 3 or *i*. This is a perfect interface condition. The thickness *t* of the interphase is then reduced (t → 0), and the interface conditions are now imperfect. This a the 2-phase problem.

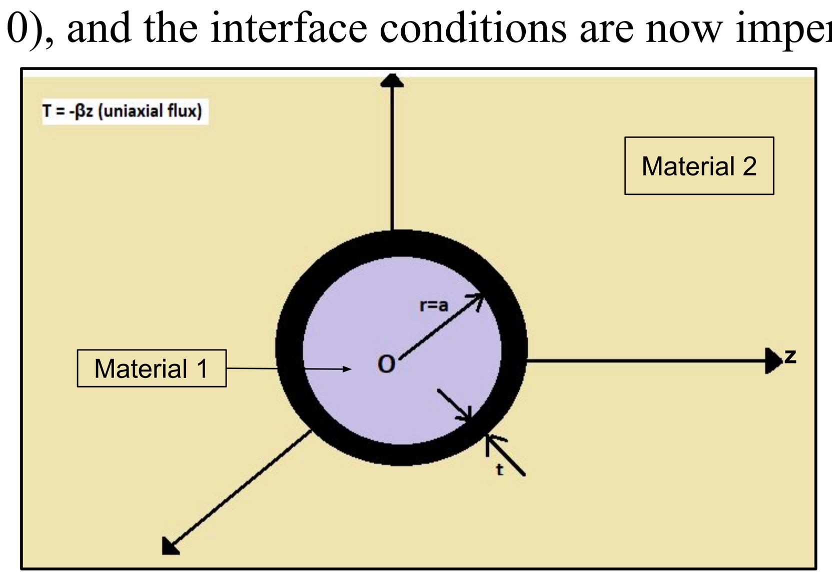

Fig. 2: 3-phase problem (perfect interface conditions)

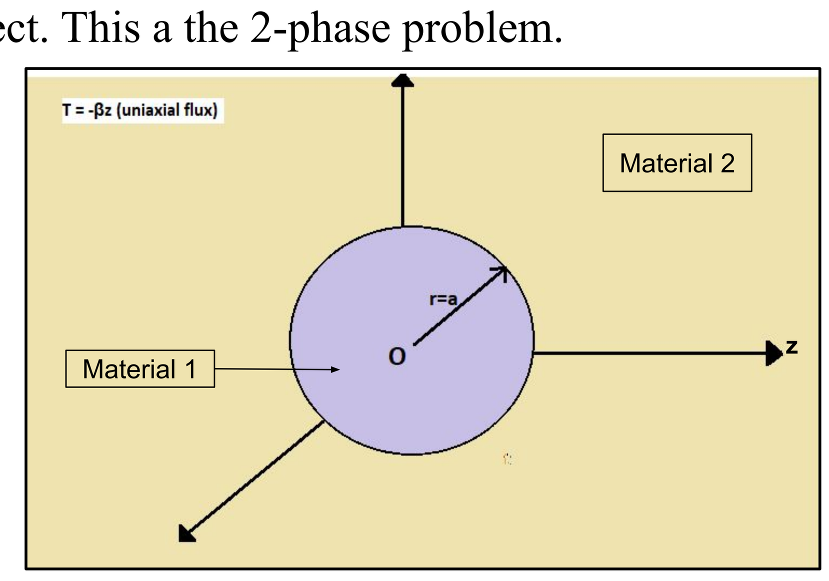

Fig. 3: 2-phase problem (imperfect interface conditions)

5. RESULTS

Temperature

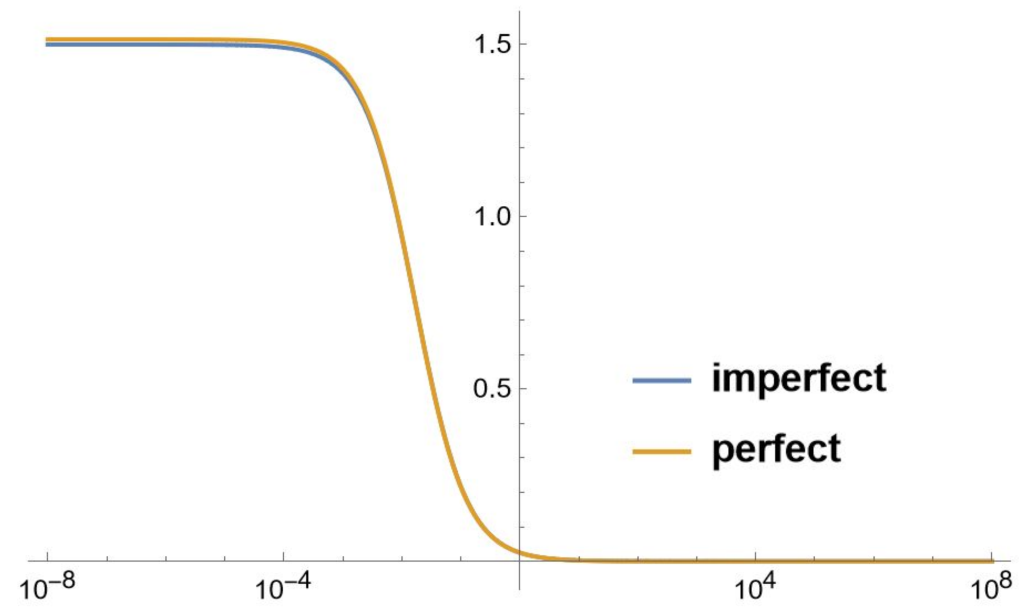

Fig. 4: Temperature jump across a perfect and an imperfect interface

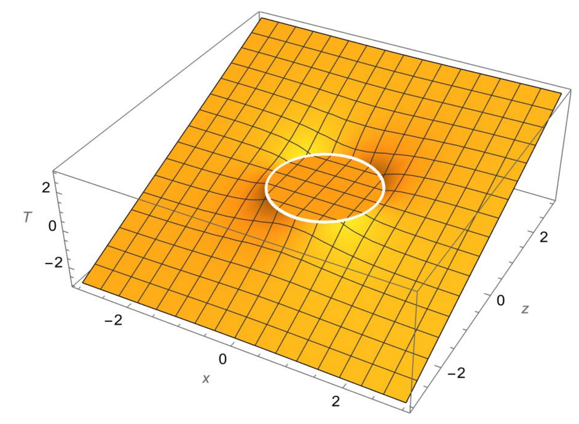

Fig. 5: Temperature jump across a perfect interface (3-phase problem)

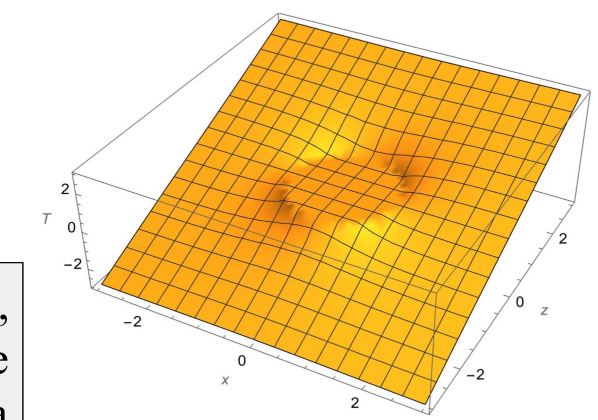

Fig. 6: Temperature jump across an imperfect interface (2-phase problem)

When the interphase conductivity is low (2-phase problem), there is a finite temperature jump but no variation in the normal flux and the coated inclusion effect is that of a homogeneous superconducting inclusion (fig 4, 5 and 6).

Heat Flux

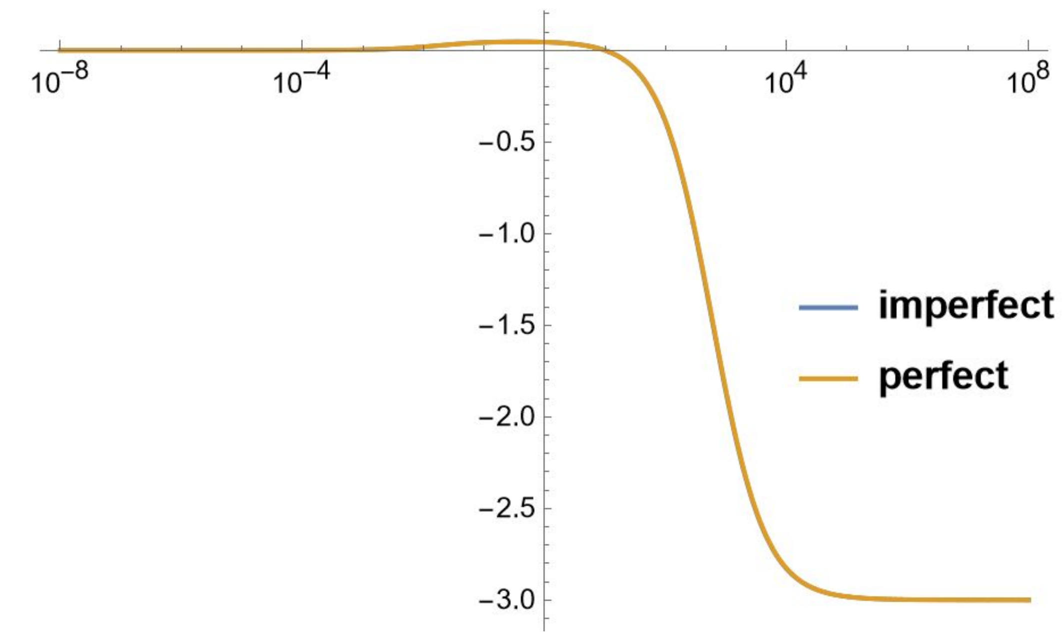

Fig. 7: Heat flux jump across a perfect interface and an imperfect interface

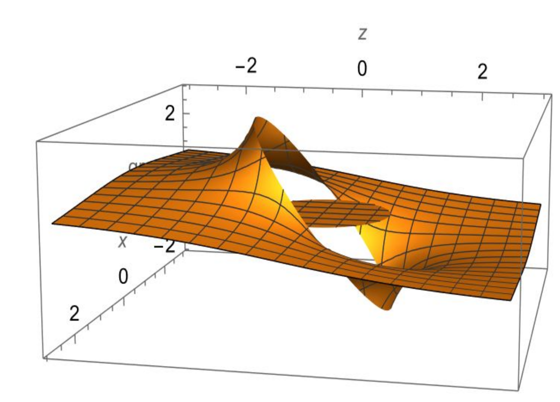

Fig. 8: Heat Flux jump across a perfect interface (3-phase problem)

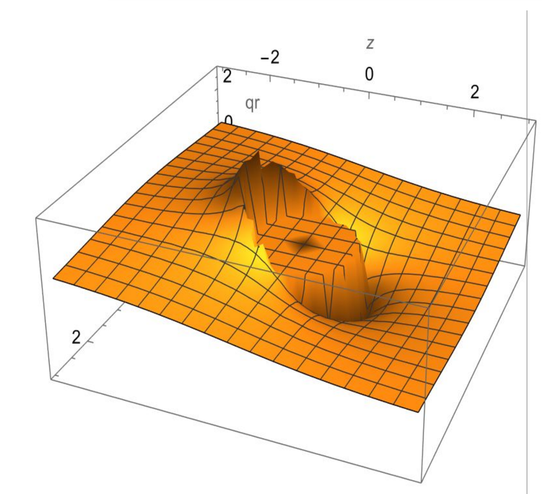

Fig. 9: Heat flux jump across an imperfect interface (2-phase problem)

When the interphase conductivity is high (3-phase problem), there is no temperature jump but a finite jump of normal flux and the coated inclusion effect is that of a homogeneous superconducting inclusion (Fig 7, 8, and 9).

6. CONCLUSION

The work presented here has resulted in establishment of the imperfect interface conditions which express the effect of a thin interphase in the context of conductivity. The present formulation is valid for the entire range of interphase conductivity. It is emphasized that the imperfect interface conditions given here apply strictly to the case of thin interphase.
The method here will be extended to derive transmission conditions for three cases: (a) Elastic media (b) Thermoelastic media (c) Thermoplastic media.

7. NEXT STEPS

Thermoelasticity: the transmission conditions across the spring type interface will be applied to a thermoelastic material. Subsequently, a new set of transmission conditions would be derived, to study the variation of properties across the interface.

Thermoplasticity: the transmission conditions for the thermoelastic material, will be extended to derive the transmission conditions across the interface of a thermoplastic materials to study the variation of properties across its interface.

8. REFERENCES

1. Hashin, Z., 2001. Thin interphase/imperfect interface in conduction. J. Appl. Phys. 89, 2261–2267.
2. Benveniste, Y., Miloh, T., 2001. Imperfect soft and sti. interfaces in two-dimensional elasticity. Mech.Mater. 33, 309–324.
3. Hashin, Z.: Thin interphase/imperfect interface in elasticity with application to coated fiber composites. J. Mech. Phys. Solids 50, 2509–2537 (2002)

ACKNOWLEGMENT

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 955944.