Transmission Conditions across a thin thermoelastic interphase

June 8th, 2023

“Modelling and optimal design of refractories for high temperature industrial applications for a low carbon society”

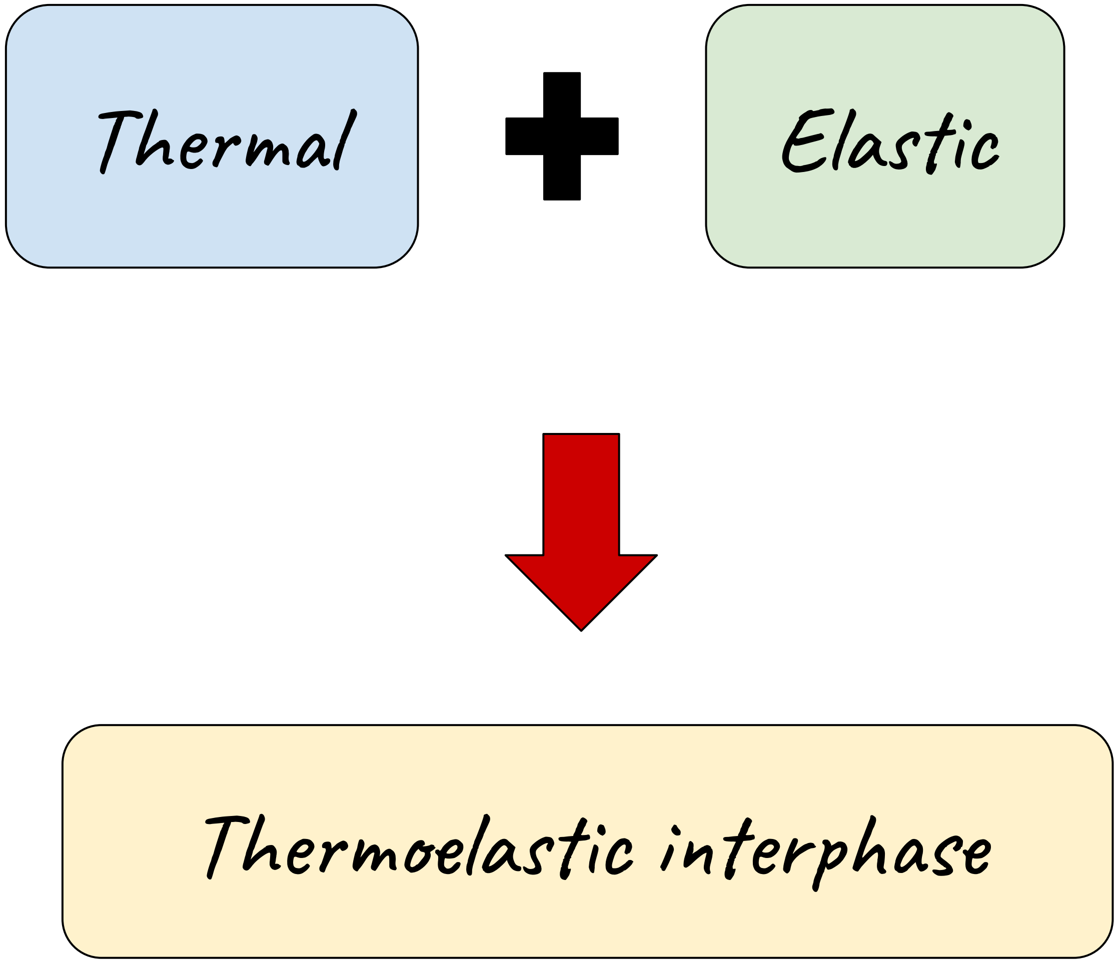
-Shubhra Pande
Andrea Piccolroaz
Severine Romero-Baivier

UNIVERSITÀ DI TRENTO
Department of Civil, Environmental and Mechanical Engineering

VESUVIUS

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 955944 – RE-FRACTURE2.

ITN EID PROJECT
RE-FRACTURE2


Thermal
Elastic
Thermoelastic interphase

Thermal Transmission Conditions

Elastic Transmission Conditions

Thermoelastic Transmission Conditions

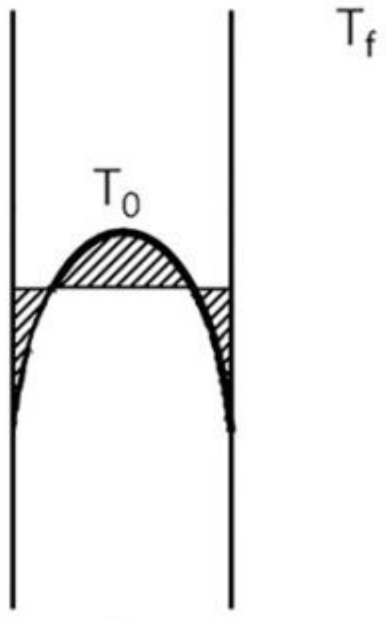
Summary and next steps

Why ?

A body subjected to thermal shock will develop tensile stresses within itself.

At micro-level, local stresses accumulate at the boundaries

Leads to crack formation.



T0 > Tf

It is important to understand the transmission conditions occuring at the micro-level

Approaches

Taylor Series Expansion

$$f_{A_2} = f_{A_1} + \left(\frac{\partial f_{A_1}}{\partial y}\right) t + \left(\frac{\partial^2 f_{A_1}}{\partial y^2}\right)\frac{t^2}{2} + O(t^3)$$

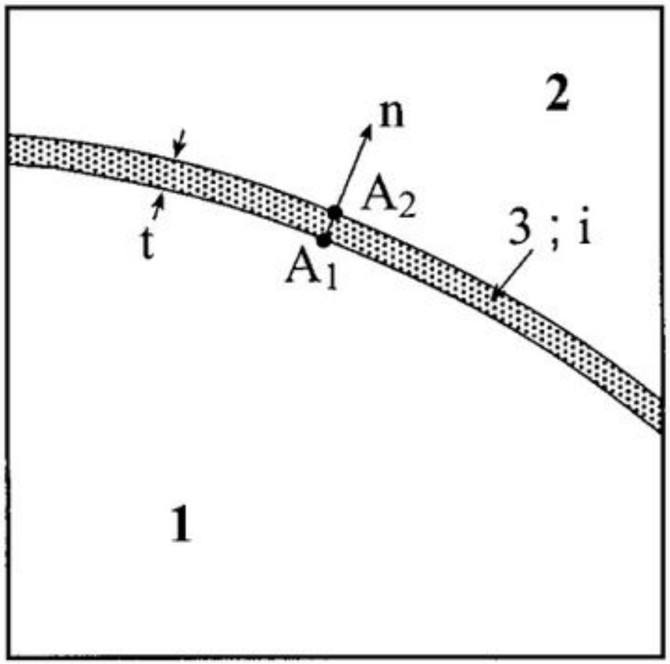


Fig.1: Thin Interphase. Image taken from the thermal paper by Hashin.

Re-scaling approach

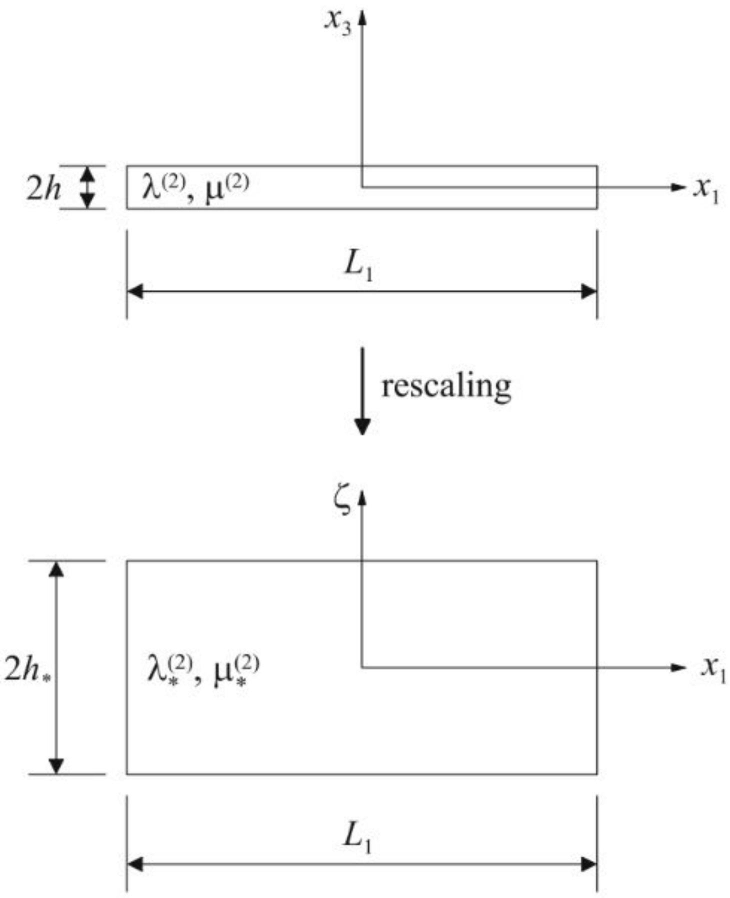
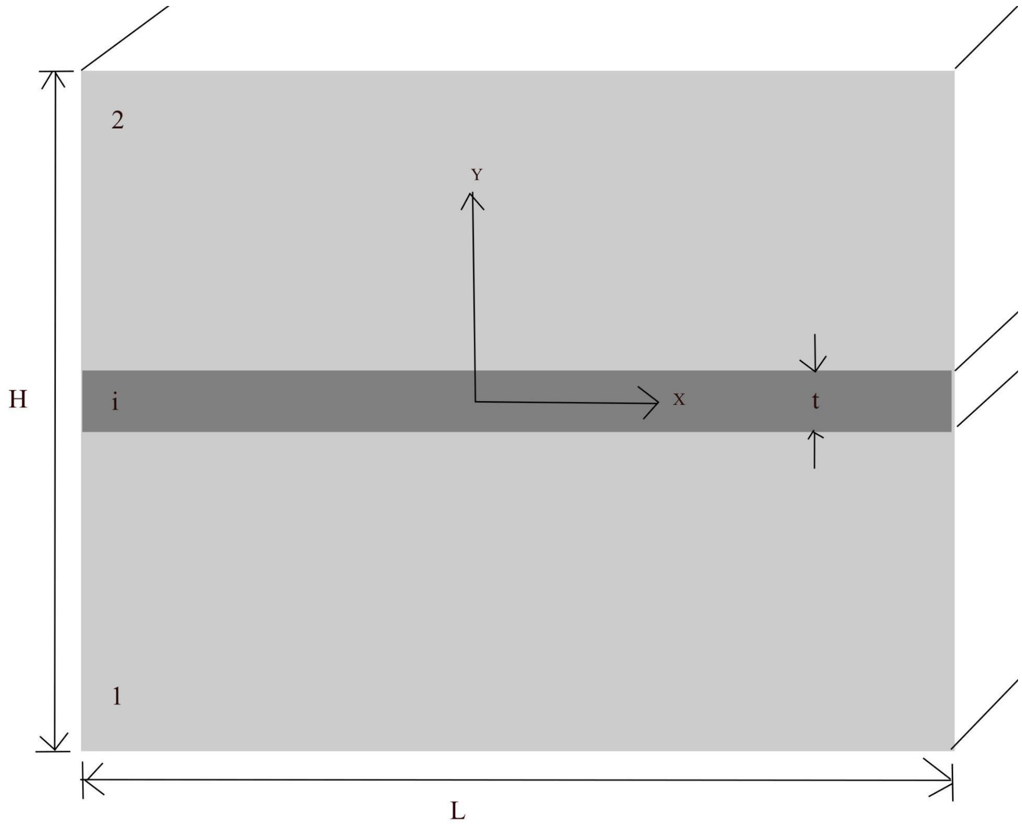


Fig.2: Image taken from the elasticity paper by Mishuris et al.

Development of Transmission Conditions for a thermal conductive interphase



GOVERNING EQUATIONS

1. Heat transfer equation: $\nabla \cdot (k \nabla T) + Q = c\rho \frac{\partial T}{\partial t}$

2. Fourier's Law: $\boldsymbol{q} = -\boldsymbol{k}(T) \nabla T$

Literature Background

Thin interphase/imperfect interface in conduction

- Z. Hashin $\rightarrow$ Taylor Series expansion

Transmission Conditions

- **Planar interface transmission conditions (without a heat source)**

$$\left(\frac{\partial T^i}{\partial \boldsymbol{n}}\right) t = [\![T]\!]$$

Jump in temperature

$$[\![q_n]\!] = k_i \nabla_s^2 T^i t$$

Jump in heat flux

$$\nabla_s^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$$

- **Transmission conditions for a planar interface with a heat source Q0:**

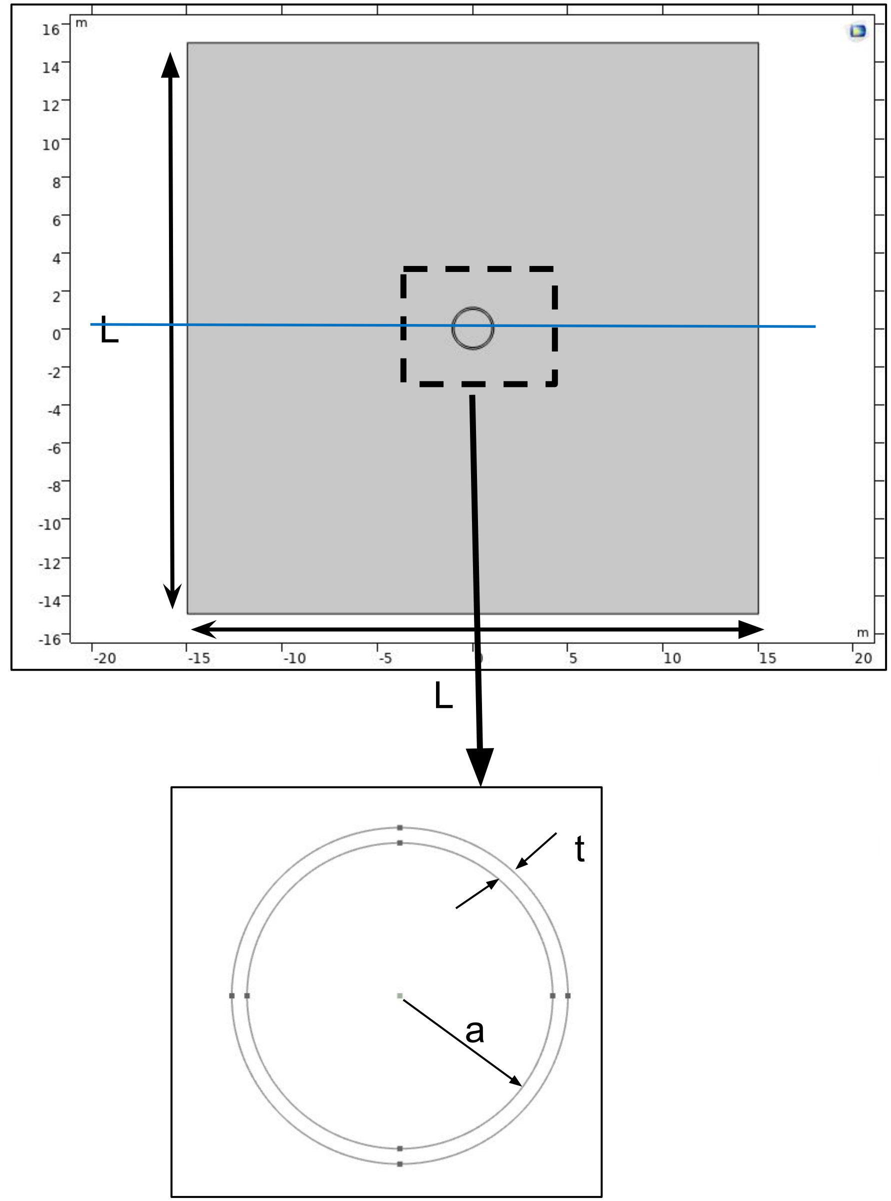
$$[\![q_n]\!] = \left(k_i \nabla_s^2 T^i + Q_0\right) t$$

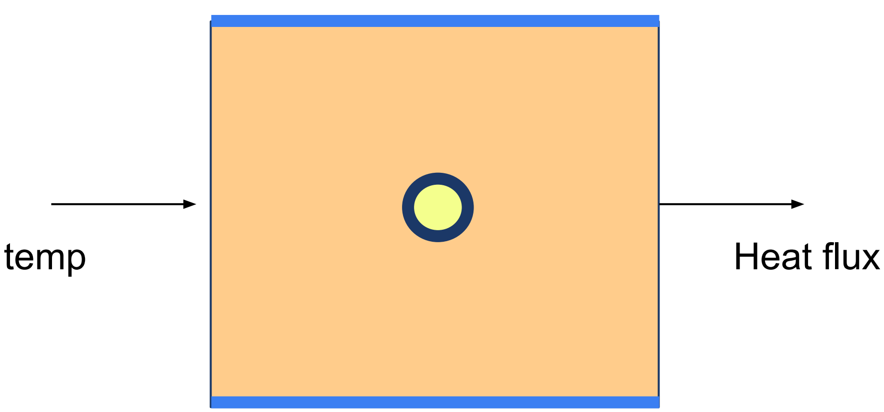
Comsol: Geometry

Label: Parameters 1

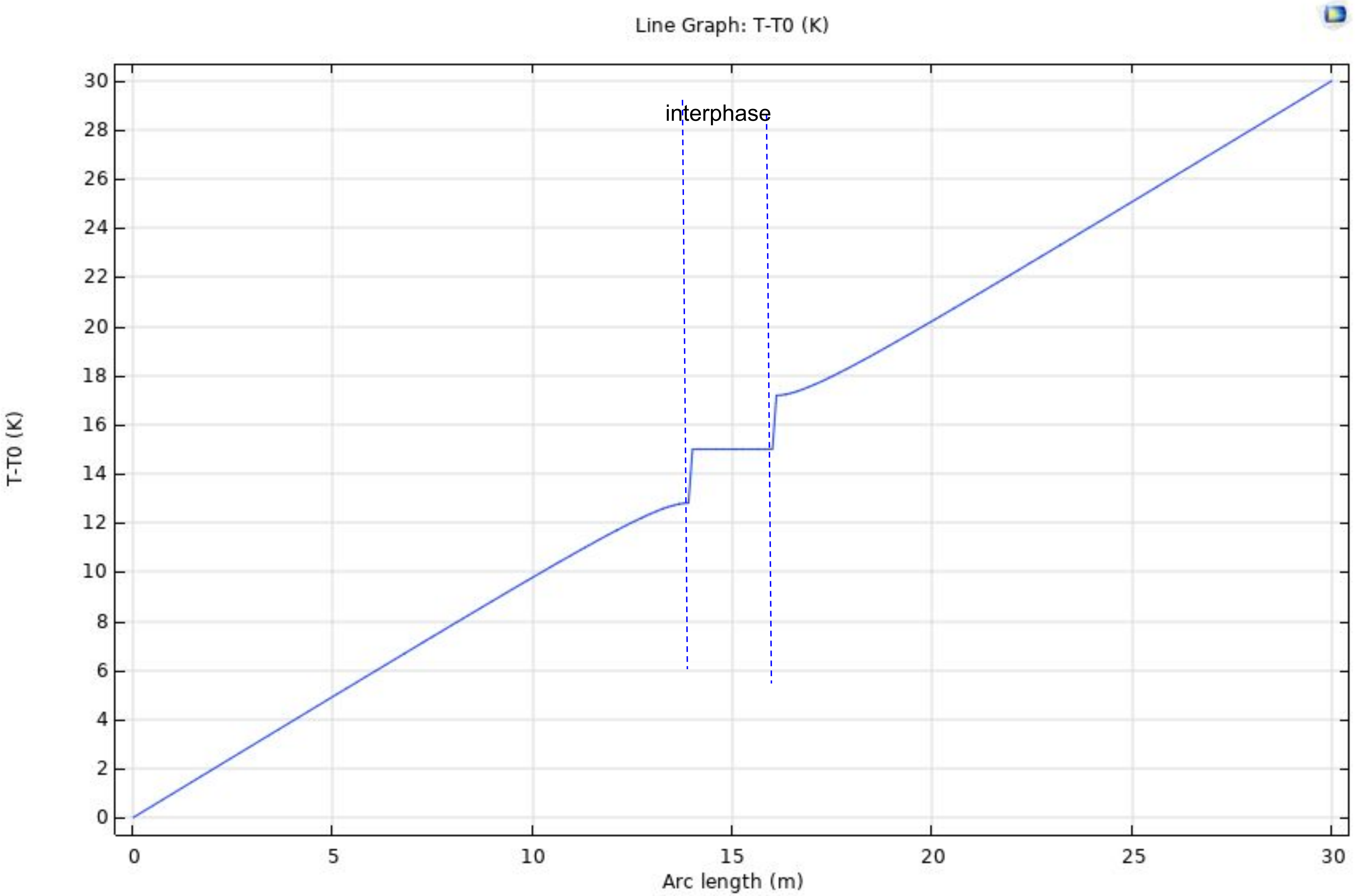
Parameters

Name	Expression	Value
L	30[m]	30 m
a	1[m]	1 m
epsilon	0.1[m]	0.1 m
k1	10[W/m/K]	10 W/(m·K)
k2	1[W/m/K]	1 W/(m·K)
ki	1e-6[W/m/K]	1E-6 W/(m·K)
beta	-1[K/m]	-1 K/m
T0	293.15[K]	293.15 K
Q0	0.001[W/m^3]	0.001 W/m³

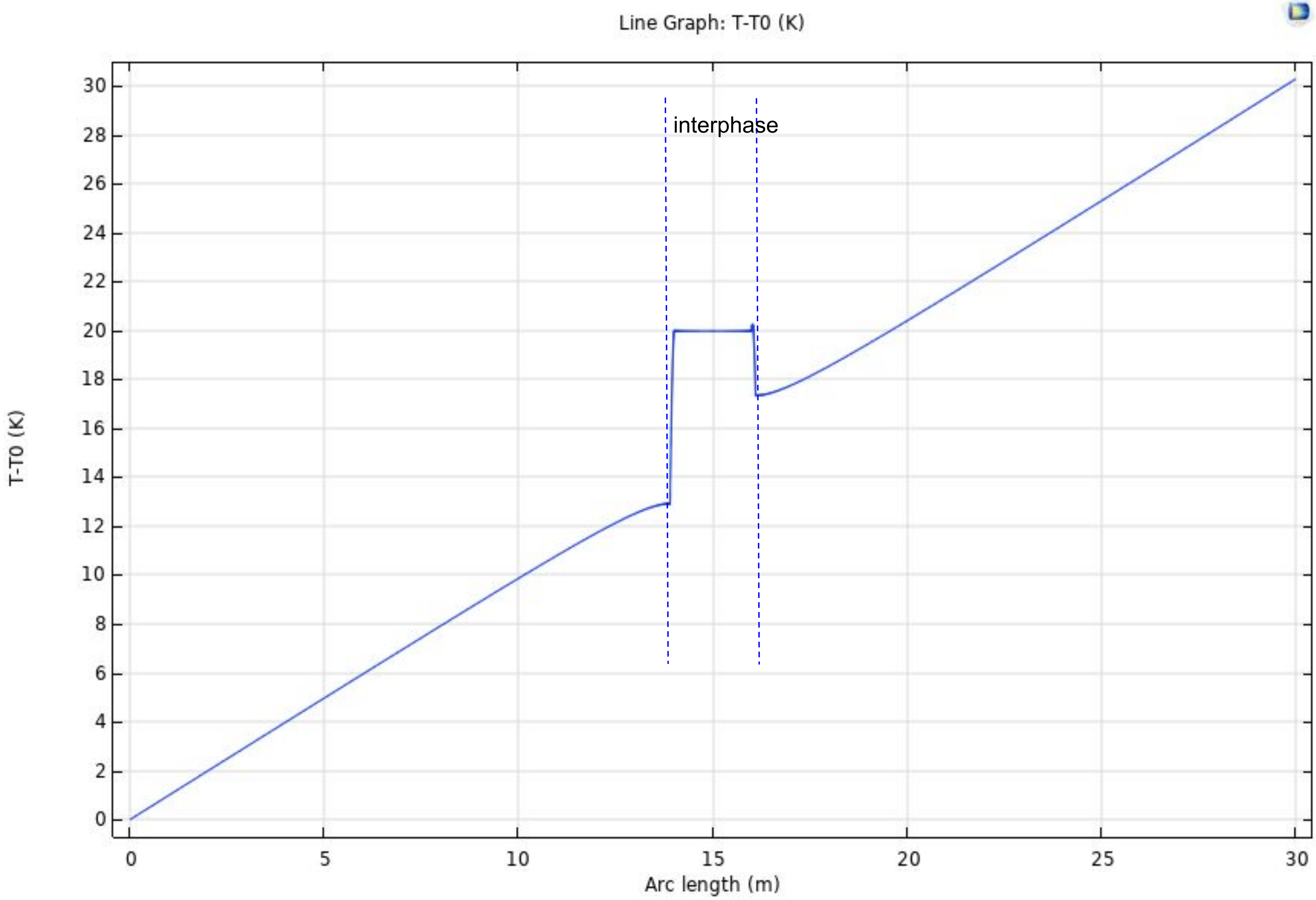




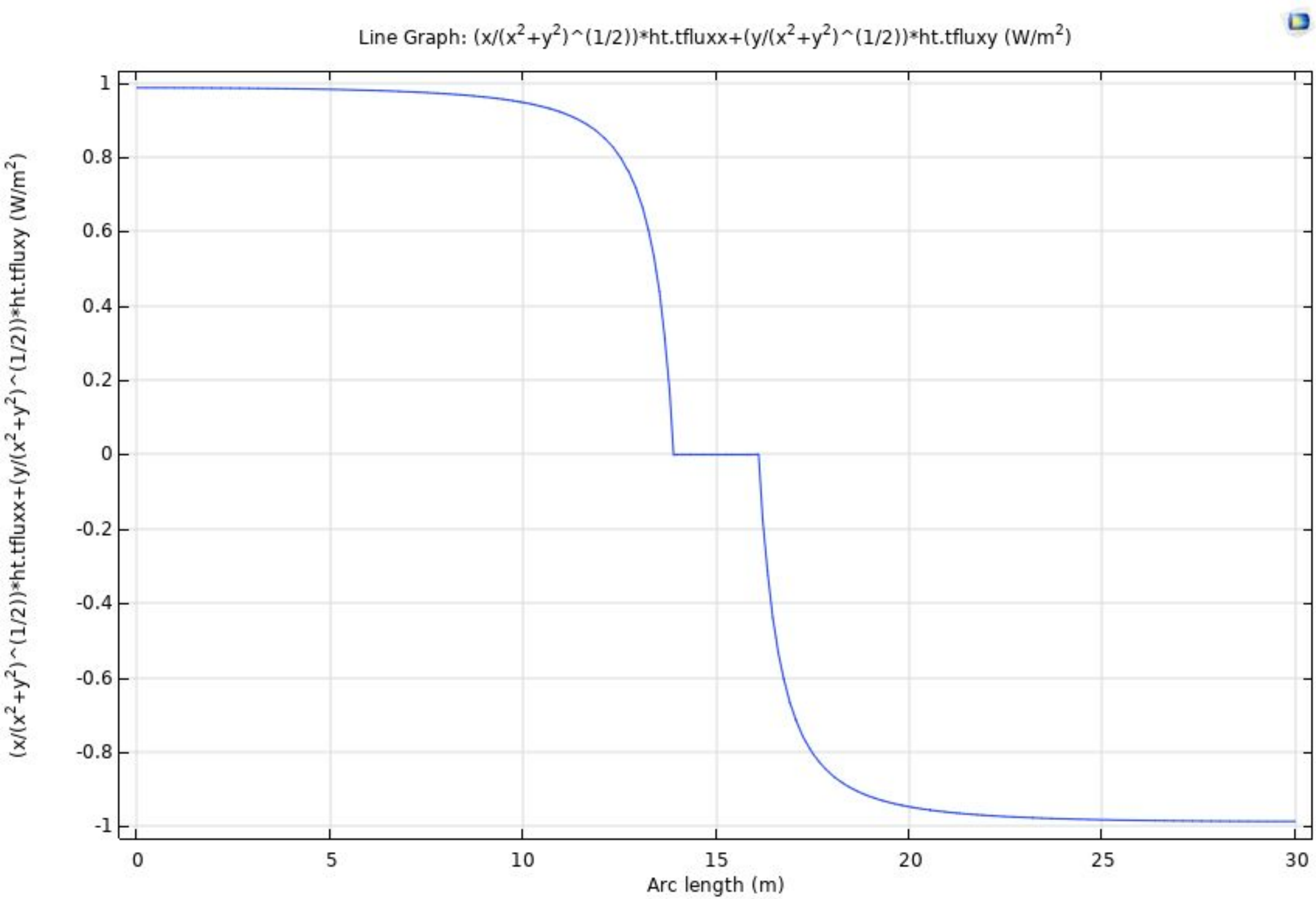
Comsol: Temperature without Heat Sources



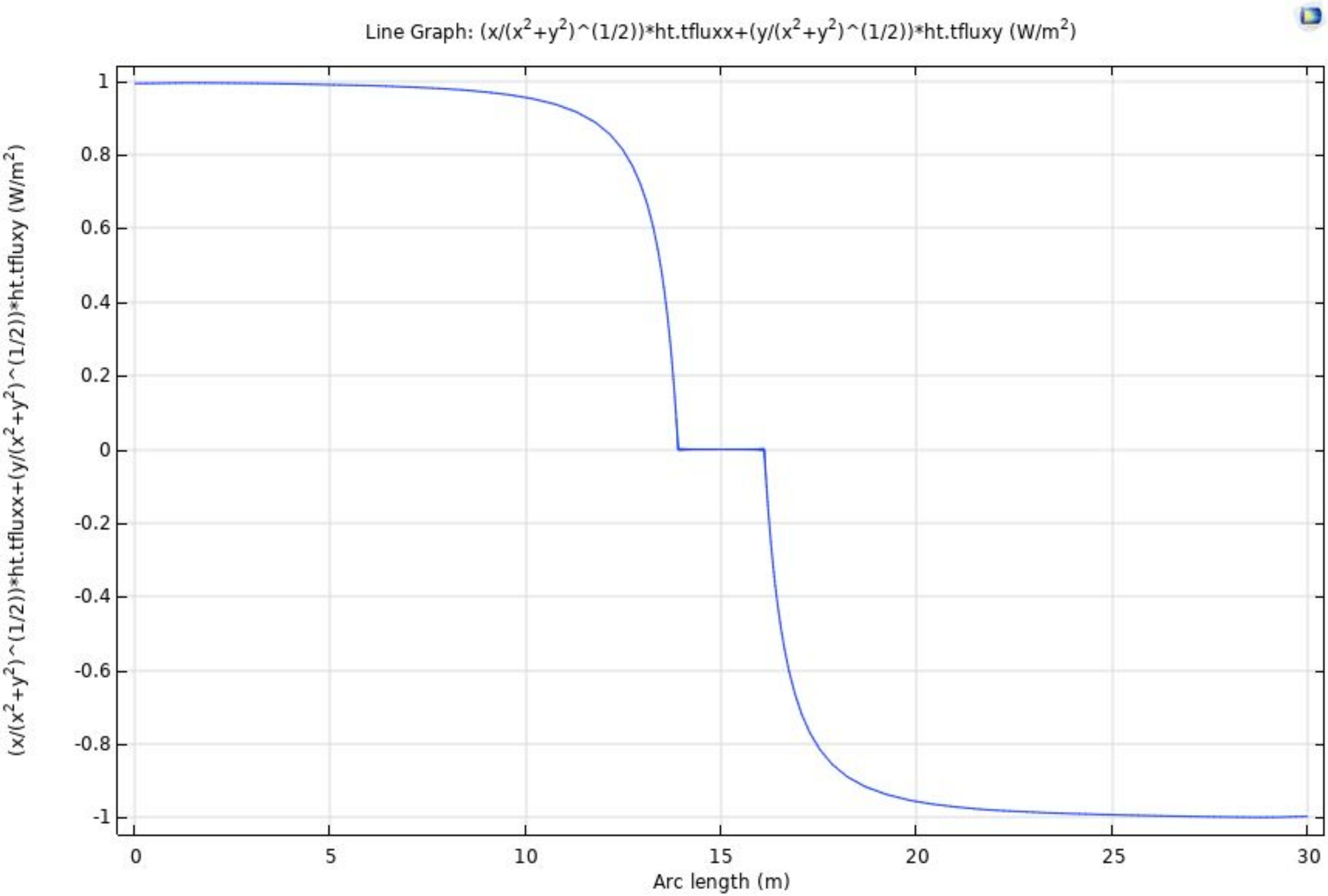
Comsol: Temperature with Heat Sources



Comsol: Heat Flux without Heat Sources



Comsol: Heat Flux with Heat Sources



Development of Transmission Conditions for an elastic “spring-type” interphase

Linear Transmission Conditions

- First transmission conditions

$$[\![u_1]\!] = [\frac{\sigma_{11}}{\lambda_i + 2G_i} - \frac{\nu_i}{1-\nu_i}(\varepsilon_{22} + \varepsilon_{33})]\,t$$

mp in the normal direction

$$[\![u_2]\!] = [\frac{\sigma_{12}}{G_i} - \frac{\partial u_1}{\partial x_2}]\,t$$

$$[\![u_3]\!] = [\frac{\sigma_{13}}{G_i} - \frac{\partial u_1}{\partial x_3}]\,t$$

- Second Transmission conditions

$$[\![\sigma_{11}]\!] = -G_i[\nabla_s^2 u_1 + \frac{\partial}{\partial x_1}\left(\frac{\partial u_2}{\partial x_2} + (\frac{\partial u_3}{\partial x_3}\right)]\,t$$

$$\nabla_s^2 \equiv \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$$

$$[\![\sigma_{12}]\!] = -\lambda_i \frac{\partial [\![u_1]\!]}{\partial x_2} - [(\lambda_i + G_i)\frac{\partial}{\partial x_2}\left(\frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}\right) + G_i \nabla_s^2 u_2]\,t$$

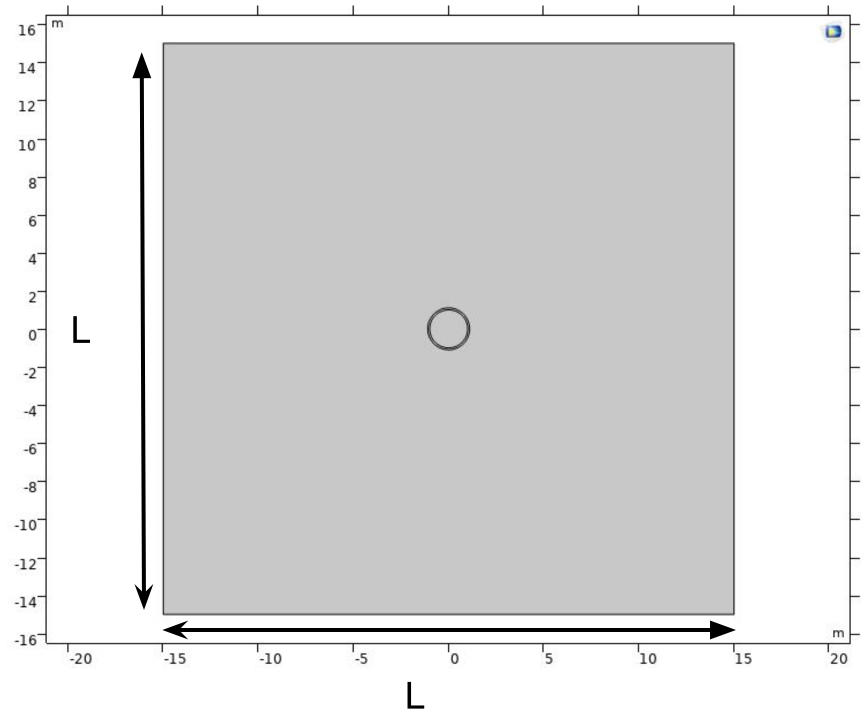
$$[\![\sigma_{13}]\!] = -\lambda_i \frac{\partial [\![u_1]\!]}{\partial x_3} - [(\lambda_i + G_i)\frac{\partial}{\partial x_3}\left(\frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}\right) + G_i \nabla_s^2 u_3]\,t$$

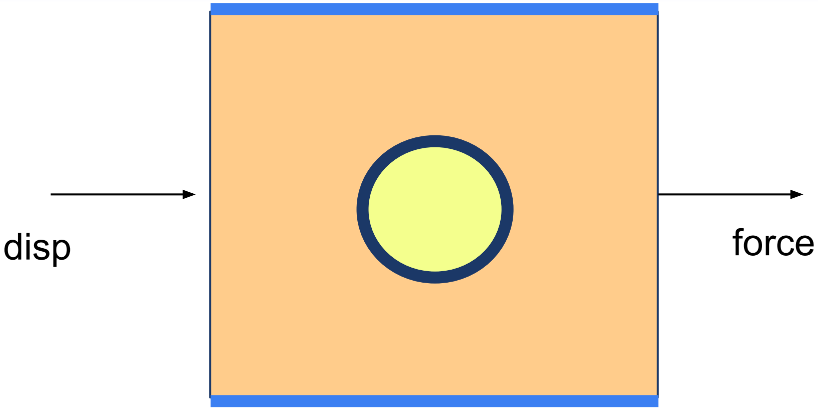
Comsol: Geometry

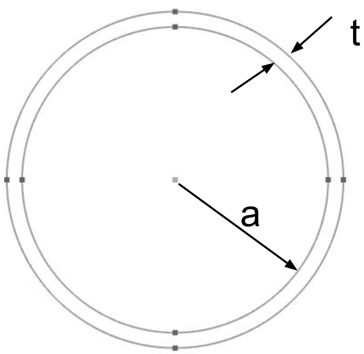
Label: Parameters 1

Parameters

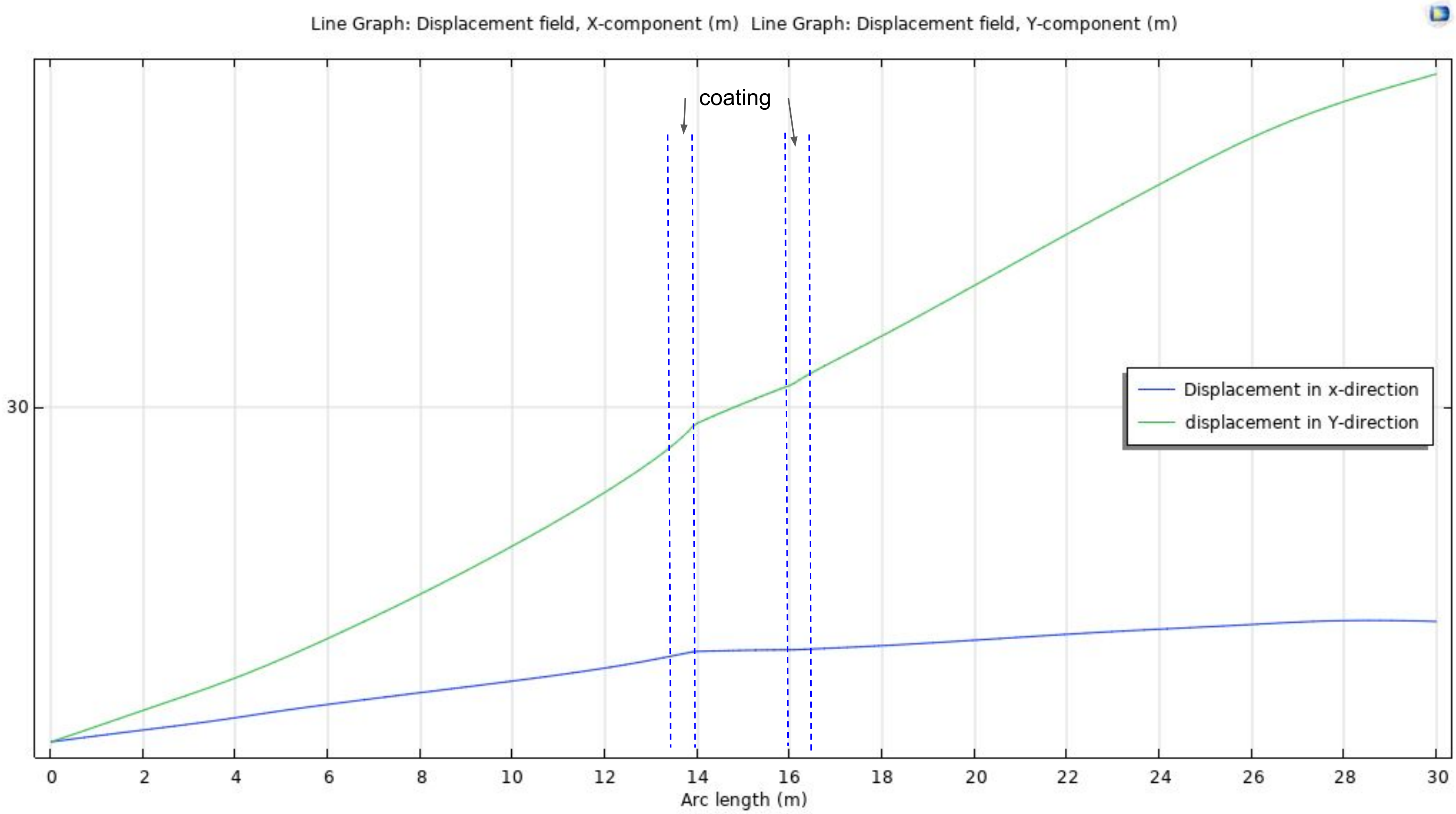
Name	Expression	Value	D
L	30[m]	30 m	
a	1[m]	1 m	
epsilon	0.1	0.1	
nu1	0.20	0.2	
nu2	0.35	0.35	
nui	0.30	0.3	
beta	1	1	
E1	24e9[Pa]	2.4E10 Pa	
E2	2.7e9[Pa]	2.7E9 Pa	
Ei	6.552e9[Pa]	6.552E9 Pa	







Comsol: Displacement



Development of Transmission Conditions for an thermoelastic interphase

UNIVERSITÀ DI TRENTO
Department of
Civil, Environmental and Mechanical Engineering

VESUVIUS

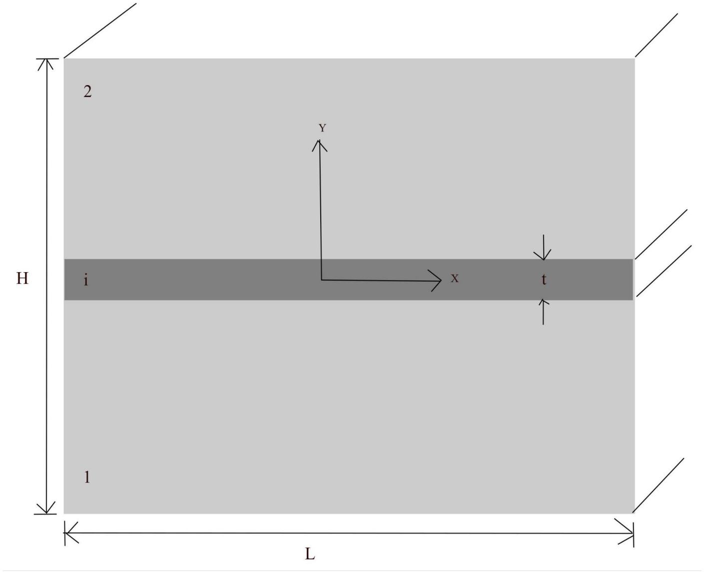
Thermoelastic Transmission Conditions

GOVERNING EQUATIONS:

1. Hooke's Law: $\boldsymbol{\sigma} = \lambda \operatorname{tr}(\boldsymbol{\varepsilon})\boldsymbol{I} + 2\mu\boldsymbol{\varepsilon} - m(T - T_0)\boldsymbol{I}$
2. Energy Equation: $\rho c_E \dot{T} = -\operatorname{div}\boldsymbol{q} + Q - mT_0 \operatorname{tr}(\dot{\boldsymbol{\varepsilon}})$

Literature Background

- Interface Models in Coupled Thermoelasticity - M. Serpilli, S. Dumont, R. Rizzoni, F. Lebon



Transmission conditions for a planar interface

Displacement

$$[\![u_y]\!] = \left(\frac{\partial u_y}{\partial y}\right) t = \left[\frac{\sigma_{yy} - \lambda_i(\varepsilon_{xx} + \varepsilon_{zz}) - m_i\theta}{2\mu_i + \lambda_i}\right] t$$

$$[\![u_x]\!] = \left(\frac{\sigma_{xy}}{\mu_i} - \frac{\partial u_y}{\partial x}\right)^i t$$

$$[\![u_z]\!] = \left(\frac{\sigma_{zy}}{\mu_i} - \frac{\partial u_y}{\partial z}\right)^i t$$

Temperature

$$[\![\theta]\!] = \theta_{A_2} - \theta_{A_1} = \left(\frac{\partial \theta}{\partial y}\right)^i t$$

Transmission conditions for a planar interface

Stresses

$$[\![\sigma_{yy}]\!] = -G_i \left[\frac{\partial}{\partial y} \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right) + \nabla_s^2 u_y \right] t + m_i t(\theta_{,x} + \theta_{,z})$$

$$[\![\sigma_{xy}]\!] = -\lambda_i \frac{\partial}{\partial x} [\![u_y]\!] + m_i t(\theta_{,x} + \theta_{,z}) - \left[(\lambda_i + G_i) \frac{\partial}{\partial x} \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) + G_i \nabla_s^2 u_x \right] t$$

$$[\![\sigma_{zy}]\!] = -\lambda_i \frac{\partial}{\partial z} [\![u_y]\!] + m_i t(\theta_{,x} + \theta_{,z}) - \left[(\lambda_i + G_i) \frac{\partial}{\partial z} \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) + G_i \nabla_s^2 u_z \right] t$$

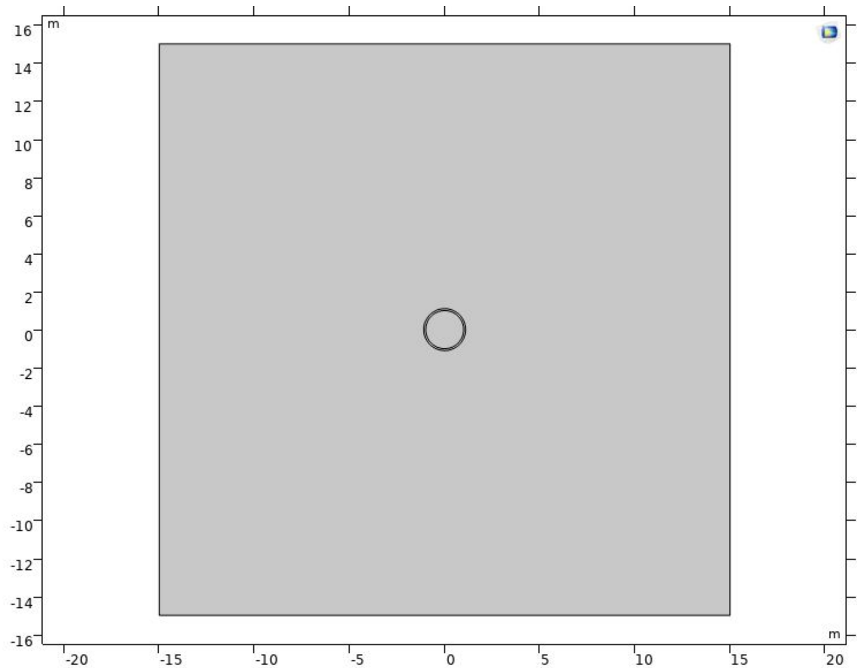
Heat Flux

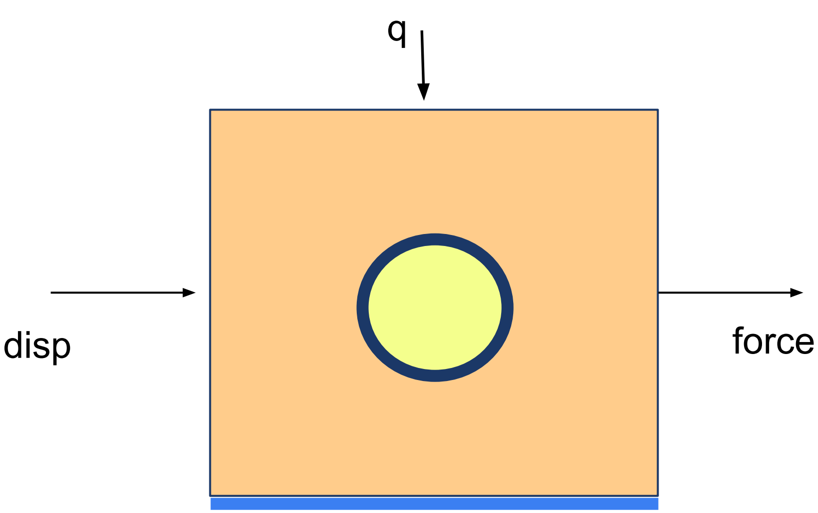
$$[\![q_y]\!] = \left[Q + m \dot{\varepsilon}_{kk} - \left(\rho c_E \dot{T} - k_i \nabla_s^2 T^i \right) \right] t$$

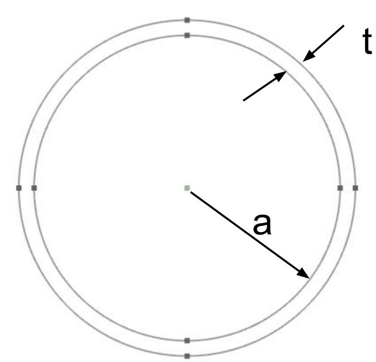
Comsol: Geometry

Parameters

Name	Expression	Value	Description
L	30[m]	30 m	Length of the domain
a	1[m]	1 m	radius of inclusion
epsilon	0.1[m]	0.1 m	thickness of the interphace
k1	10[W/m/K]	10 W/(m·K)	thermal conductivity of...
k2	1[W/m/K]	1 W/(m·K)	thermal conductivity of in...
ki	1e-6[W/m/K]	1E-6 W/(m·K)	thermal conductivity of in...
betaTH	-1[K/m]	-1 K/m	
T0	293.15[K]	293.15 K	ambient temperature
Q0	0.001[W/m^3]	0.001 W/m³	heat source inside the int...
nu1	0.20	0.2	PR for matrix
nu2	0.35	0.35	PR for inclusion
nui	0.30	0.3	PR for interphase
E1	24e9[Pa]	2.4E10 Pa	YM for matrix
E2	2.7e9[Pa]	2.7E9 Pa	YM for inclusion
Ei	6.552e9[Pa]	6.552E9 Pa	YM for interphase
betaEL	-1	-1	







Next Steps

Rescale approach for the thermoelastic transmission conditions. → Use benchmark problems to verify the TCs. → Derive thermoplastic transmission conditions.

UNIVERSITÀ DI TRENTO
Department of
Civil, Environmental and Mechanical Engineering

Thank you